



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

resulting expression is $(-1)^n \lambda^n [(n+1)a - \lambda]$ and is of the form demanded by our theorem by the case of $n+1$. The left-hand member of the resulting expression may be written so that its determinant is of order $n+1$,

$$\frac{na + a - \lambda}{na - \lambda} \cdot \begin{vmatrix} -\lambda, & 0, & 0, & \cdots, & 0 \\ a, & a - \lambda, & a, & \cdots, & a \\ a, & a, & a - \lambda, & \cdots, & a \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ a, & a, & a, & \cdots, & a - \lambda \end{vmatrix} \quad (1)$$

Here the $-\lambda$ of our multiplier appears as the element in the first row and first column of the determinant of order $n+1$. Add the second, third, and each of the following rows in turn to the first row obtaining an equal expression, the new determinant being exactly the same as that in (1) above except that the elements of the first row are now all $na - \lambda$. Multiply the factor preceding this determinant into the elements of the first row obtaining a third determinant which is again exactly like (1) except that the elements of the first row are now all $na + a - \lambda$. Subtract the second, third, and each of the following rows from the first, thus obtaining a determinant of order $n+1$ and of precisely the form demanded by our theorem if it is to be true in the case of $n+1$. Thus, if the relation is true for n , it is also true for $n+1$, and the induction is complete.

Also solved by A. L. CANDY, P. J. DA CUNHA, L. D. HAND, A. M. HARDING, R. A. JOHNSON, L. C. MATHEWSON, H. L. OLSON, A. PELLETIER, J. L. RILEY, G. Y. SOSNOW, ELIJAH SWIFT, and C. C. YEN.

2744 [1919, 37]. Proposed by E. B. ESCOTT, Chicago, Ill.

An insurance company computes its quarterly premiums by adding 6 per cent to the annual premium and dividing by 4. If a policyholder pays quarterly, what rate of interest is he paying?

I. SOLUTION BY ELIJAH SWIFT, University of Vermont.

If we assume that this means that the policyholder sets aside the annual premium at the beginning of the year, pays the first of the quarterly premiums out of it, lets the remainder lie at interest for three months, then deducts the second premium, and so on, the interest will be compounded quarterly, and the present worth of the four premiums at the beginning of the year must equal the annual premium. If we call the annual premium, $4P$, and the (unknown) annual interest rate, $4i$, each quarterly premium will be $1.06P$ and we have the equation

$$1.06P + \frac{1.06P}{1+i} + \frac{1.06P}{(1+i)^2} + \frac{1.06P}{(1+i)^3} = 4P.$$

This cubic may be solved by Horner's method, whence $4i = 16.11$ per cent.

If interest be compounded semi-annually, we have a quadratic,

$$1.06P + \frac{1.06P}{1+i} + \frac{1.06P}{1+2i} + \frac{1.06P}{(1+i)(1+2i)} = 4P$$

whence $4i = 16.33$ per cent.

If interest be reckoned as simple (compounded annually) we have to solve the cubic

$$1.06P + \frac{1.06P}{1+i} + \frac{1.06P}{1+2i} + \frac{1.06P}{1+3i} = 4P.$$

whence $4i = 16.54$ per cent.

In any case, then, the policyholder must pay over 16 per cent for the accommodation.

II. SOLUTION BY THE PROPOSER.

A (by algebra).

Let P = annual premium, p = quarterly premium = $\frac{1.06}{4}P$, r = annual rate of interest.

$$P = \frac{400}{106} p.$$

We have the equation

$$(P - p) \left[1 + \frac{r}{4} \right]^3 - p \left[1 + \frac{r}{4} \right]^2 - p \left[1 + \frac{r}{4} \right] - p = 0.$$

Putting

$$1 + \frac{r}{4} = x$$

and substituting value of P , we have the equation

$$294x^3 - 106x^2 - 106x - 106 = 0.$$

Solving by Horner's Method, we have

$$x = 1 + \frac{r}{4} = 1.04028,$$

$$r = .16112 = 16.112 \text{ per cent compounded quarterly.}$$

B (by arithmetic). A more elementary and more "practical" method is the method by trial and error. A few trials will show that the rate is something over 16 per cent.

First Trial. Taking the rate as 16 per cent and the annual premium as 100, we have the scheme,—

Annual premium due.....	100.00
First quarterly premium paid.....	26.50
	<hr/>
	73.50
Interest for three months.....	2.94
	<hr/>
	76.44
Second quarterly premium paid.....	26.50
	<hr/>
	49.94
Interest for 3 months.....	2.00
	<hr/>
	51.94
Third quarterly premium paid.....	26.50
	<hr/>
	25.44
Interest for 3 months.....	1.02
	<hr/>
	26.46
Fourth quarterly premium.....	26.50
	<hr/>
First error.....	0.04

We see that 16 per cent is slightly too small.

Second trial. Taking the rate as 16.2 per cent., we have, in the same way as before, an error of + .03.

Forming a table

Rate	Error
16	-.04
16.2	+.03

By interpolation, the rate that will give zero error is

$$16 + \frac{4}{7} \times .2 = 16.114 \text{ per cent.}$$

If greater accuracy were required, repeat the computation with the last rate and interpolate again.

Also solved by G. N. ARMSTRONG, H. N. CARLETON, and H. L. OLSON.

2747 [1919, 72]. Proposed by DANIEL KRETH, Wellman, Iowa.

In the right triangle ABC , right angle C , we have given on the hypotenuse the segments $AD = 15$, $DE = 10$, $EB = 15$, and the angle DCE equal to the angle ECB . Find the angle DCE , and the sides AC and BC .